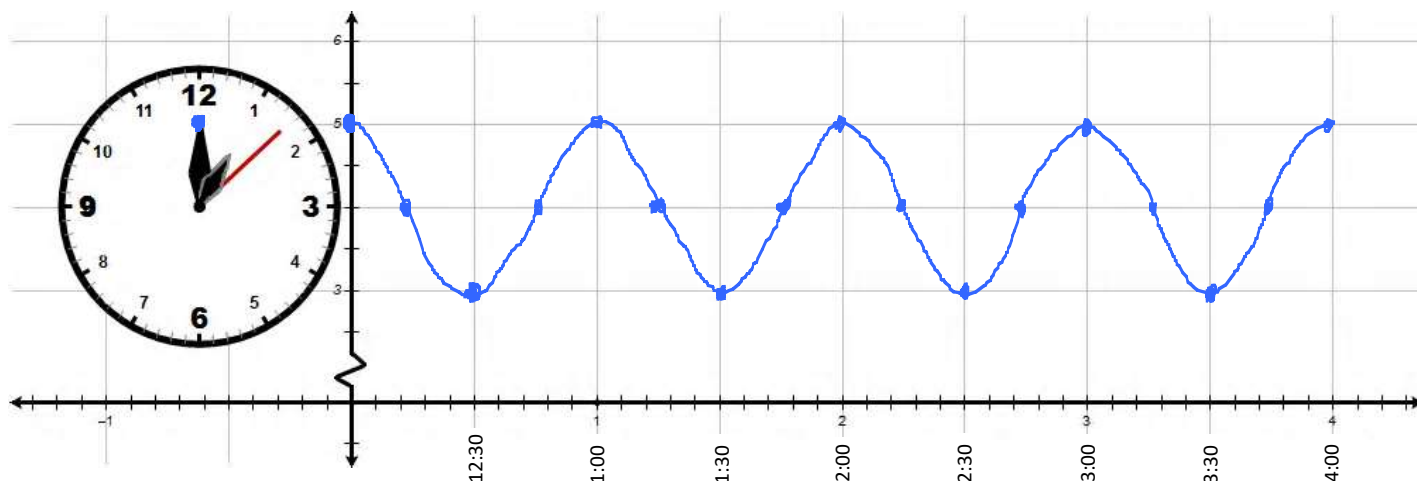
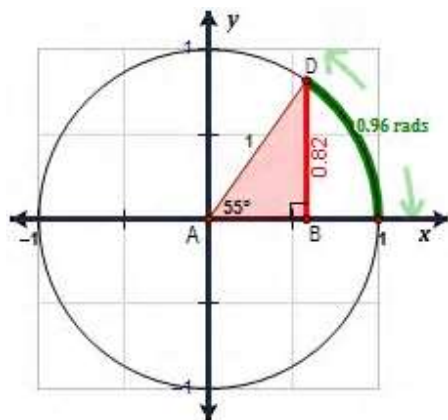


Let's consider a large clock that is hung on a wall such that the center of the clock is exactly 4 feet above the ground. The tip of the hour hand moves between a maximum of 5 feet and a minimum of 3 feet off the ground. Create a plot of the height of the tip of the hour hand starting from 12:00 am to 4:00 am. (The picture of the clock shows a time of roughly 1:00 am)

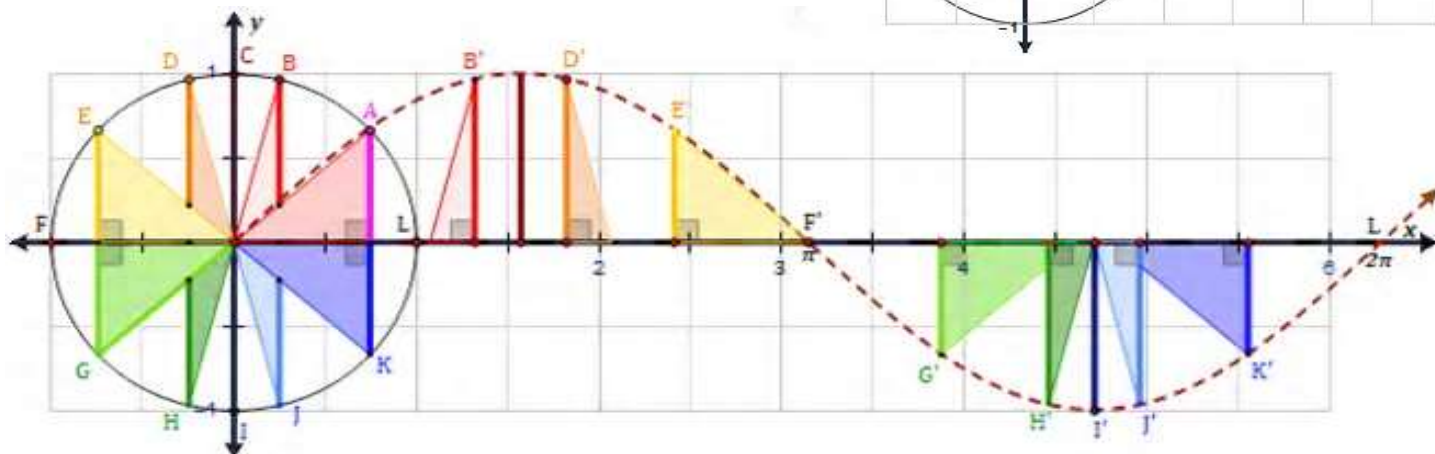
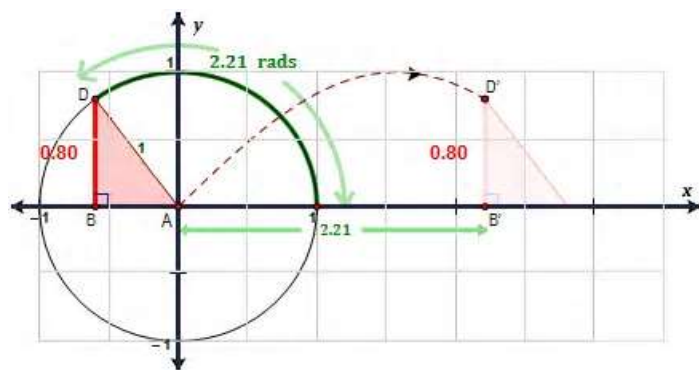


This plot should generate a **cosine wave** graph (see the animation on Matt's Math Labs). Hopefully, this should make you ask, "How do functions that represent ratios of triangles generate smooth waves?"



How Right Triangles create a WAVE: First consider a right triangle drawn inside of a unit circle like the one shown at the right. Determine the value of $\sin(A)$. Try the example at the right find $\sin(55^\circ)$ which is approximately the same as finding $\sin(0.96 \text{ rads})$.

Then, if we plot each of point where x is the measure of the angle of the unit circle in radians and y is value of $\sin(A)$ which is essentially the height of the right triangle for the given angle. The resulting locus of points generates a Sine Wave. The diagram below shows several more right triangles that map out more of the sine wave.

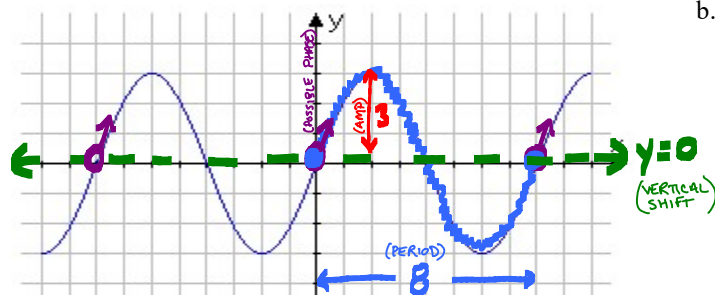


A Sine Wave is commonly described by 4 components.

Parts of a Sine Wave	Visual
Amplitude: The amplitude of a wave is the distance from the midline to highest point of the wave. (half of the vertical displacement). $\text{Amplitude} = \frac{(\text{Highest } Y - \text{Lowest } Y)}{2}$	
Period: The period of the wave is the horizontal distance of one complete cycle. $\text{Period} = (\text{Crest } X) - (\text{Previous Crest } X)$ $\text{Period} = (\text{Trough } X) - (\text{Previous Trough } X)$	
Phase Shift: The amount the wave is shifted right or left $\text{Phase Shift} = \frac{(\text{Crest } X) + (\text{Previous Trough } X)}{2}$	
Vertical Shift: The amount the wave is shifted up or down $\text{Vertical Shift} = \frac{(\text{Highest } Y + \text{Lowest } Y)}{2}$	

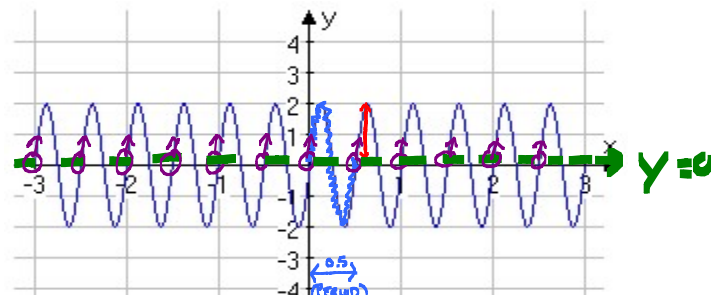
1. Find a possible equation of the following graphs.

a.



Amplitude = 3
 Period = 8
 Phase Shift = 0
 Vertical Shift = 0

b.

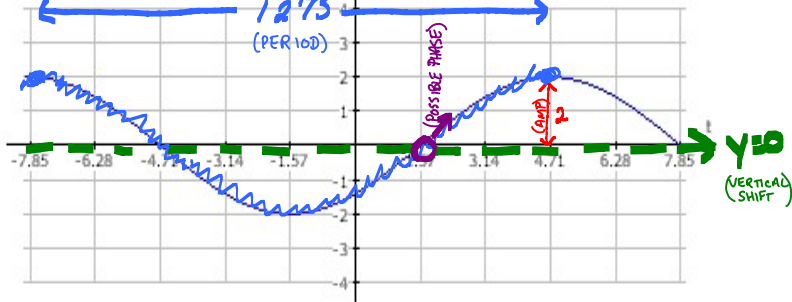


Amplitude = 2
 Period = 0.5
 Phase Shift = 0
 Vertical Shift = 0

2. Find a possible equation of the following graphs.

a. $\text{PERIOD} = \text{CREST } x - \text{PREVIOUS CREST } x$

$$= 4.71 - -7.85$$



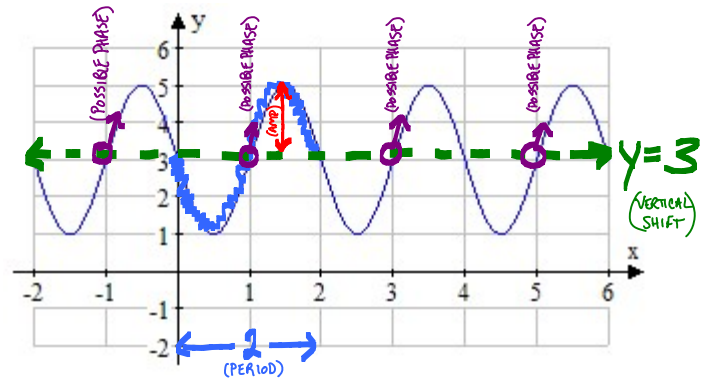
Amplitude = 2

Period = 12.57

Phase Shift = 1.57

Vertical Shift = 0

b.



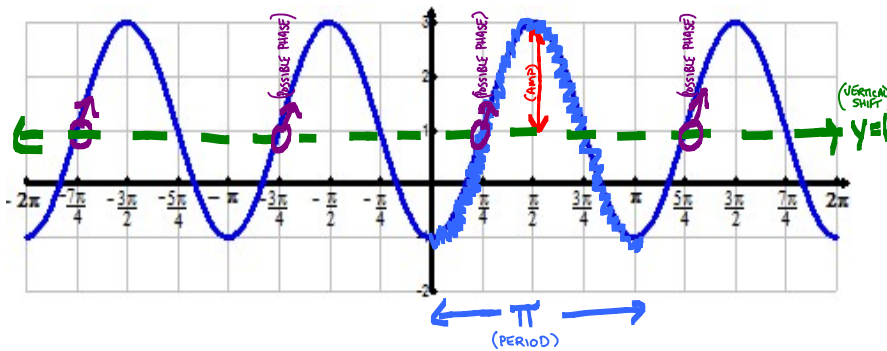
Amplitude = 2

Period = 2

Phase Shift = -1 or 1

Vertical Shift = 3

c.



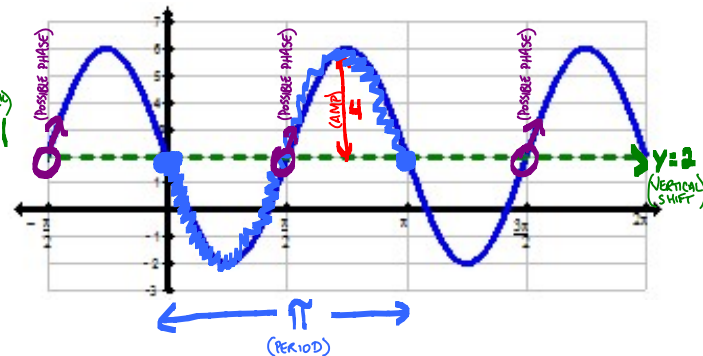
Amplitude = 2

Period = π

Phase Shift = $\frac{\pi}{4}$

Vertical Shift = 1

d.



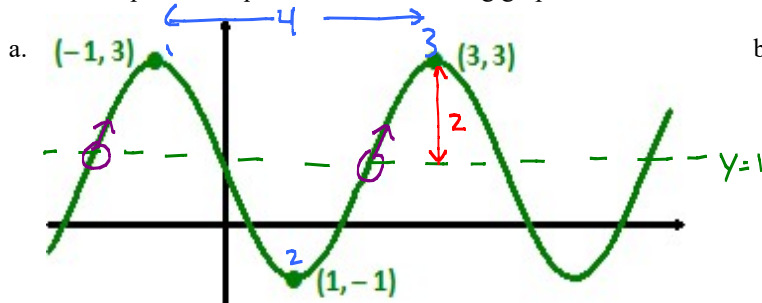
Amplitude = 4

Period = π

Phase Shift = $-\frac{\pi}{2}$ or $\frac{\pi}{2}$

Vertical Shift = 2

3. Find a possible equation of the following graphs.



VERTICAL SHIFT: $\frac{HIG + LOW}{2} = \frac{3 + (-1)}{2} = \frac{2}{2} = 1$

AMP: $\frac{HIG - LOW}{2} = \frac{3 - (-1)}{2} = \frac{4}{2} = 2$

PERIOD: CREST $X_3 - \text{CREST } X_1 = 3 - (-1) = 4$

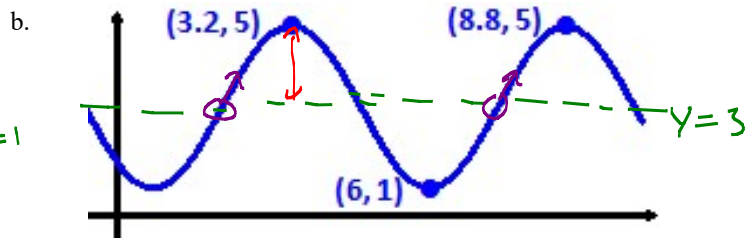
PHASE: $\frac{X_2 + X_3}{2} = \frac{1 + 3}{2} = \frac{4}{2} = 2$

Amplitude = 2

Period = 4

Phase Shift = 2

Vertical Shift = 1



VERTICAL SHIFT: $\frac{HIG + LOW}{2} = \frac{5 + 1}{2} = \frac{6}{2} = 3$

AMPLITUDE: $\frac{HIG - LOW}{2} = \frac{5 - 1}{2} = \frac{4}{2} = 2$

PERIOD: CREST $X_3 - \text{CREST } X_1 = 8.8 - 3.2 = 5.6$

PHASE: $\frac{X_2 + X_3}{2} = \frac{6 + 8.8}{2} = \frac{14.8}{2} = 7.4$

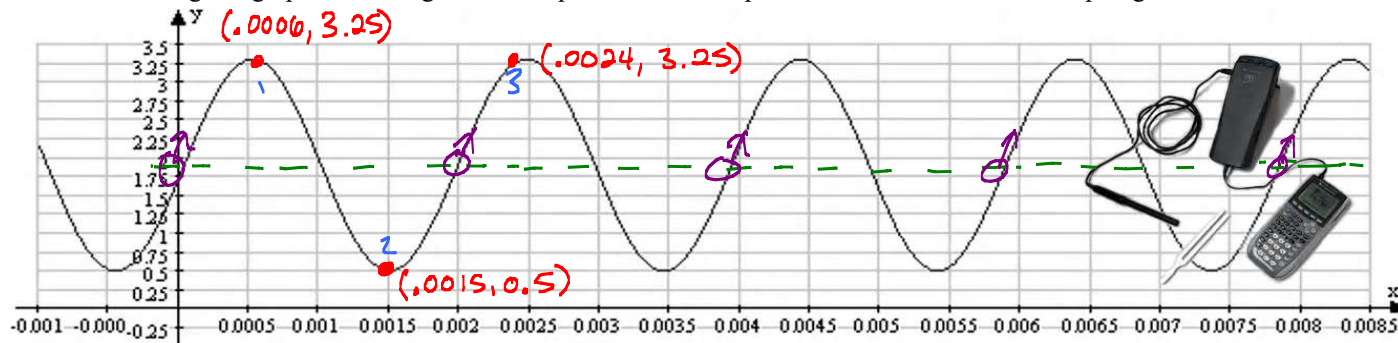
Amplitude = 2

Period = 5.6

Phase Shift = 7.4

Vertical Shift = 3

4. The following is a graph of a tuning fork held up to a CBL microphone connected to a TI-84 Graphing Calculator.



a. What is the amplitude?

1.375

AMP: $\frac{HIG - LOW}{2} = \frac{3.25 - 0.5}{2}$

b. What is the Vertical Shift?

1.875

V. SHIFT: $\frac{HIG + LOW}{2} = \frac{3.25 + 0.5}{2} = \frac{3.75}{2} = 1.875$

c. What is the Period?

.0018

PERIOD: CREST $X_3 - \text{CREST } X_1 = 0.0024 - 0.0006 = .0018$

d. What is a possible Phase Shift?

.00195

PHASE: $\frac{X_2 + X_3}{2} = \frac{0.0015 + 0.0024}{2} = \frac{0.0039}{2} = .00195$

e. What is a possible equation of the sound wave?

$y = 1.375 \sin\left(\frac{2\pi}{.0018} (x - .00195)\right) + 1.875$